Aarhus University

ZKBoo: Faster Zero-Knowledge for Boolean Circuits

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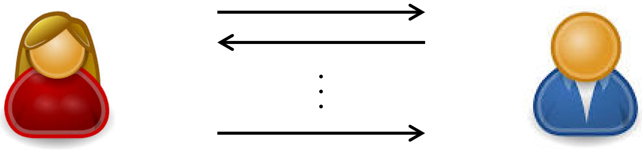
Usenix Security Symposium 2016

Zero-Knowledge (ZK) Arguments

Alice **Bob**



Private Input : $\mathbf{x}$

"I know $\mathbf{x}$ such that $\mathbf{y} = C(\mathbf{x})$"

(C and $\mathbf{y}$ public)

Output:
"yes! / no!"

In theory...

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- signature schemes 

- user identification protocols 

- electronic voting systems 

- verifiable delegation of computation 
- electronic payment system
- ... ...

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  [Gro10, Lip12, GGPR13, Lip 13, DFGK14, GRo 15]
  [PGHR13, BCGTV13, BCTV14, CTV15, CFH$^+$15]

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- ZKGC (*zero-knowledge from garbled circuits*)
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  - proving time is decreased :-)
  - interaction is required :-(

In practice...

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New!

- **ZKBoo** (*Zero-Knowledge for Boolean circuits*)
  - can be made non interactive :-)
  - fast in proving and verifying :-)
  - the size of the proof grows linearly with the circuit size :-|

Comparison for $C = \text{SHA-1}$

"I know **x** such that **y** = SHA-1(**x**)"

	Preproc. (ms)	Prover (ms)	Verifier (ms)	Proof size (B)
ZKBoo	0	**13**	**5**	454840
ZKGC*	0	> 19	> 25	186880
Pinocchio**	9754	12059	8	**288**

* estimates for the proof size and lower-bounds for the runtime

**[Parno-Howell-Gentry-Raykova 2013]

In the rest of this talk:

1. Description of the ZKBoo protocol
2. Implementation results

Σ-Protocol

Public data: $C : \{0,1\}^n \to \{0,1\}^m$ (boolean circuit) and $\mathbf{y} \in \{0,1\}^m$

Input: $\mathbf{x}$ s.t. $C(\mathbf{x}) = \mathbf{y}$

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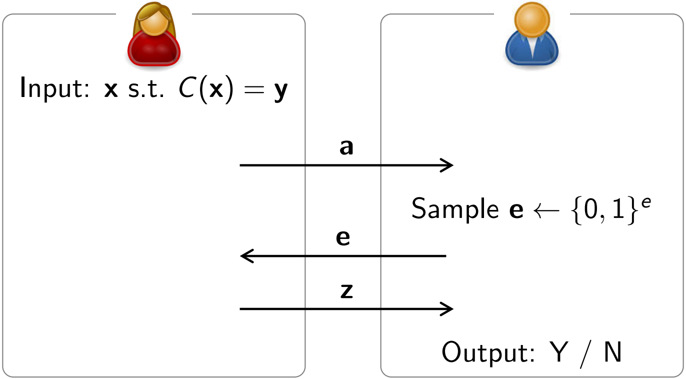

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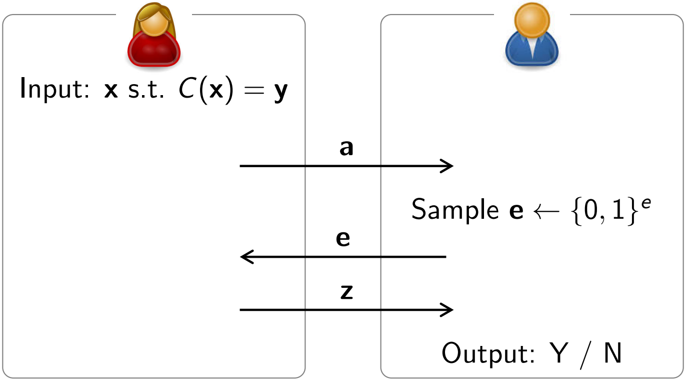

Complete: if Alice and Bob honest and $C(\mathbf{x}) = \mathbf{y}$,
$\Pr[\text{Bob outputs Y}] = 1$

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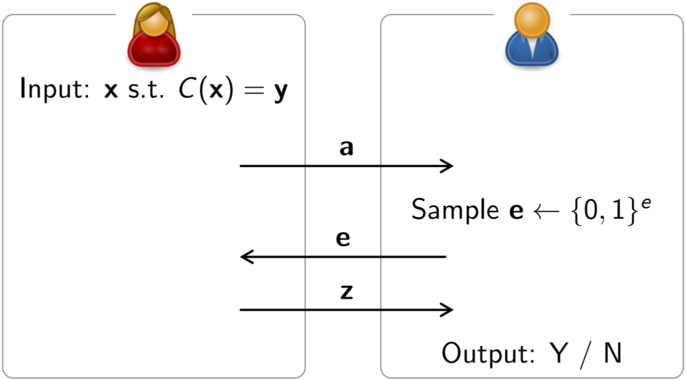

Soundness: from ≥ 2 accepting conversations $(\mathbf{a}, \mathbf{e}_i, \mathbf{z}_i)$ with $\mathbf{e}_i \neq \mathbf{e}_j$ we can efficiently compute $\mathbf{x}'$ s.t. $C(\mathbf{x}') = \mathbf{y}$

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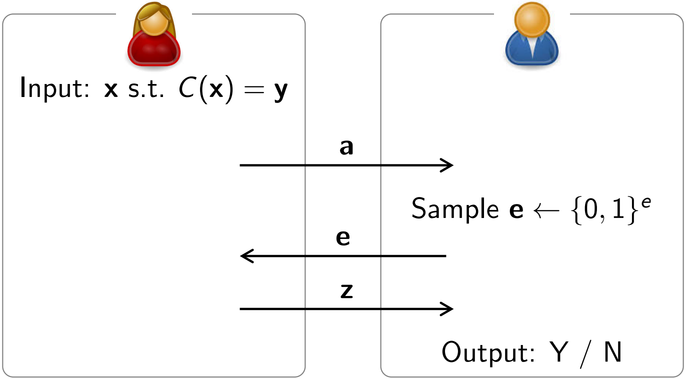

The protocol has **soundness error** ϵ:
if Alice is cheating, then Pr[Bob outputs Y] $\leq \epsilon$

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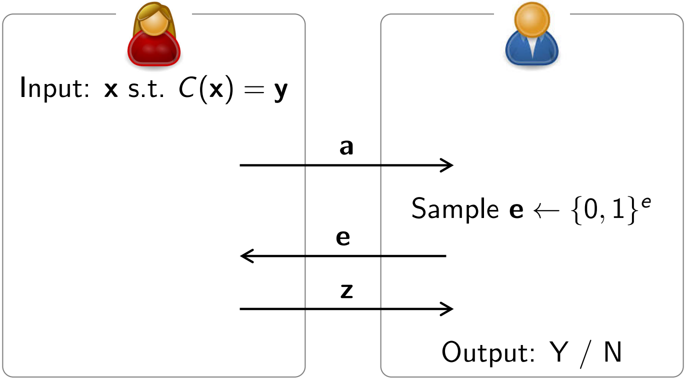

(Honest-Verifier) **ZK property**:
the distribution of $(\mathbf{a}, \mathbf{e}, \mathbf{z})$ does not reveal info on $\mathbf{x}$

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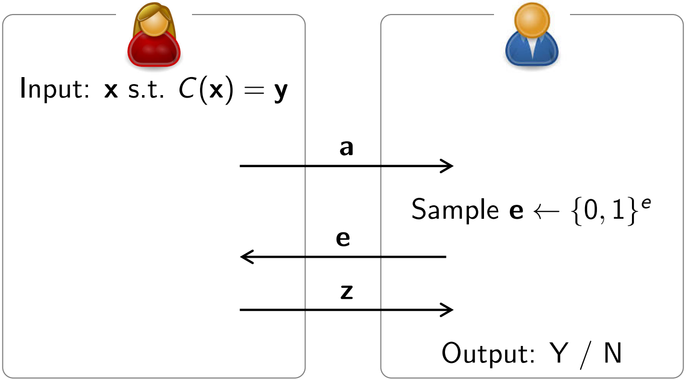
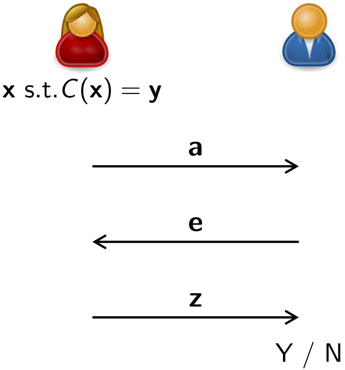


It can be made non-interactive!
(Fiat-Shamir heuristic)

Σ-Protocol Recap



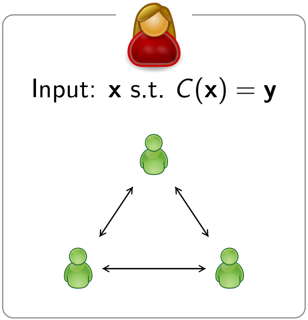
- Complete:
  if Alice honest, $\Pr[\text{Bob says Y}] = 1$
- Soundness error:
  if Alice cheats, $\Pr[\text{Bob says Y}] \leq \epsilon$
- ZK property: no info on **x**!
- 3 rounds, public coin $\rightarrow$ non-interactive

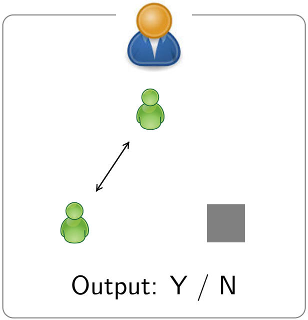
Related work:

IKOS Construction

(or “MPC-in-the-head”)

[Ishai-Kushilevitz-Ostrovsky-Sahai 2007]





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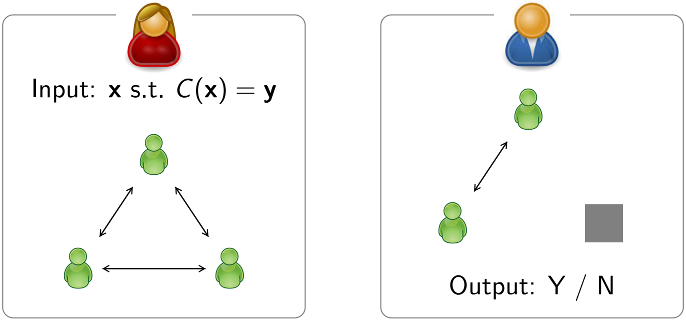

- a Σ-protocol with error $2/3$ (not implemented!)
- ZK protocol with *asymptotically* good complexity;

Circuit decomposition:

Goal: compute $C(\mathbf{x})$ splitting the computation in 3 branches s.t. looking at any 2 consecutive branches gives no info on $\mathbf{x}$

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Let N be a fixed integer, consider the following finite set of functions:

Share, Rec and

$$\mathcal{F} = \{f_1^{(j)}, f_2^{(j)}, f_3^{(j)}\}_{j=1,\ldots,N}$$

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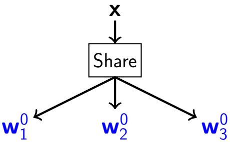

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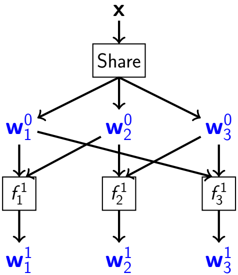

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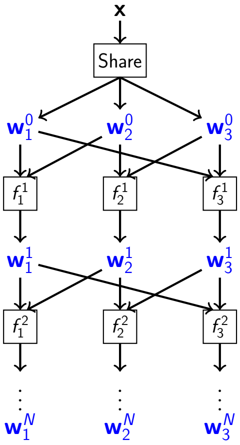

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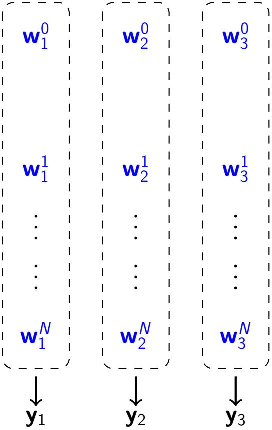

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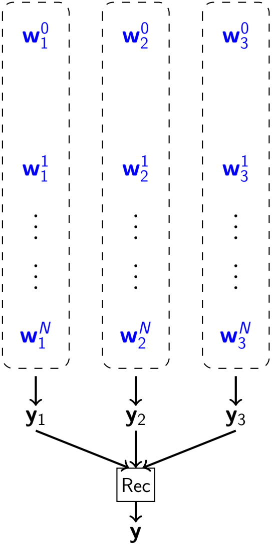

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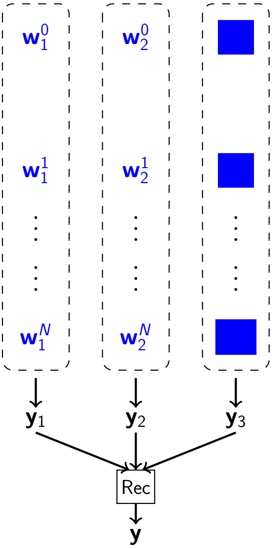

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$$\mathcal{F} = \{f_1^{(j)}, f_2^{(j)}, f_3^{(j)}\}_{j=1,\ldots,N}$$

- correctness: $\mathbf{y} = C(\mathbf{x})$
- 2-privacy: $\forall\, e, \forall\, j$ $(\mathbf{w}_e^j, \mathbf{w}j_{e+1}, \mathbf{y}_{e+2})$ doesn't reveal info on $\mathbf{x}$

ZKBoo Protocol

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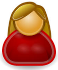

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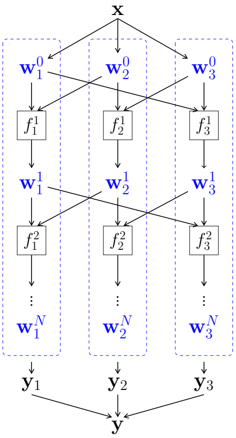

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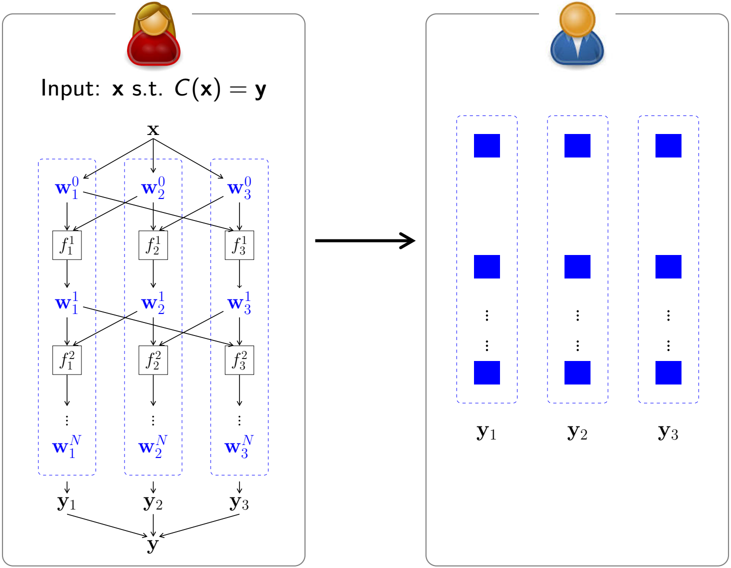

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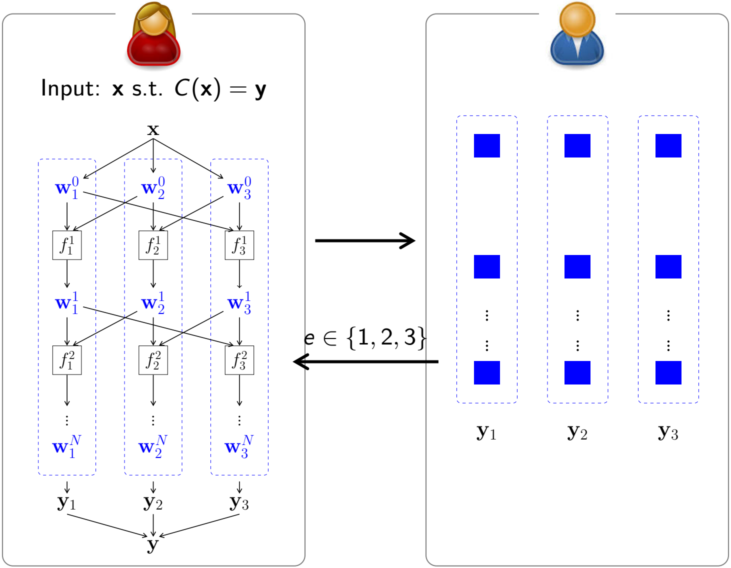

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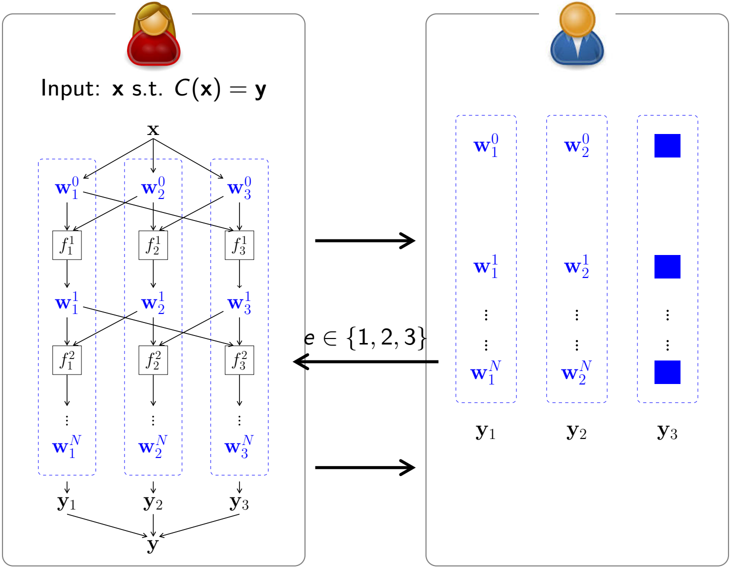

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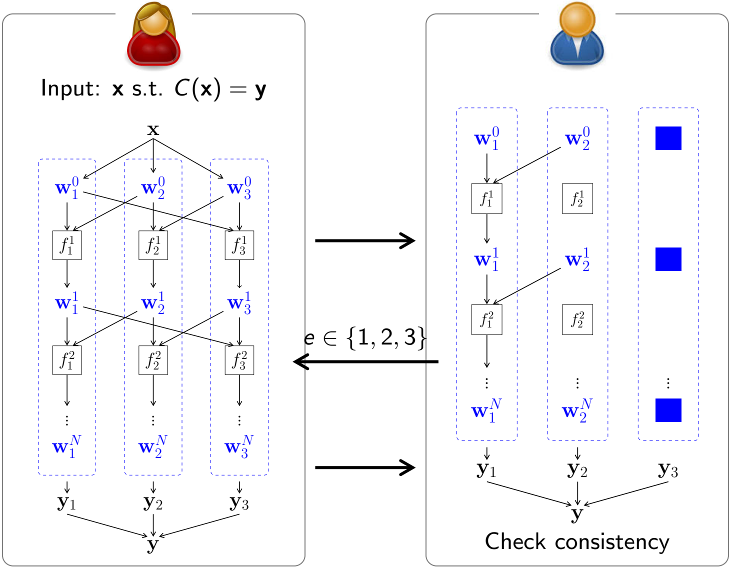

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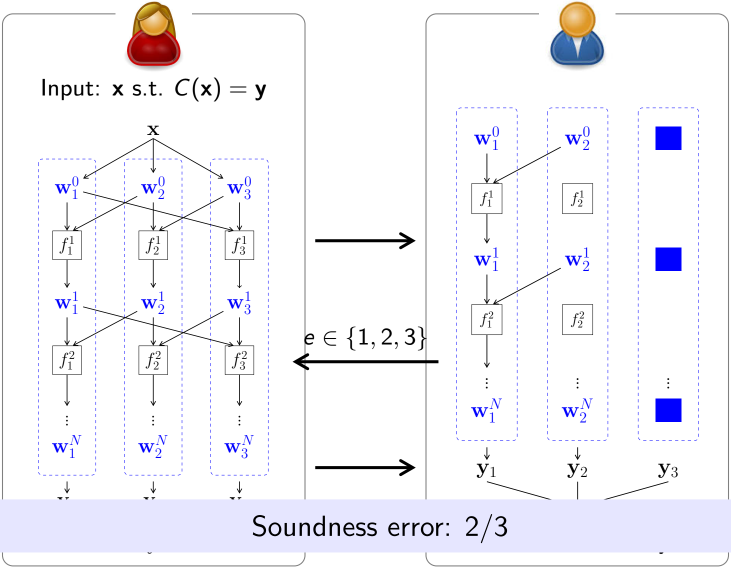

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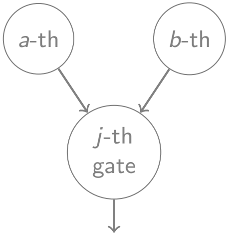

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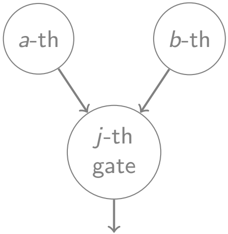

XOR gate

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$e = 1, 2, 3$

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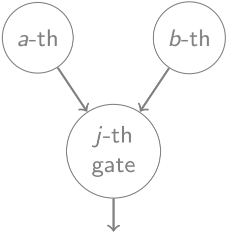

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AND gate
$f_e^{(j)}(\mathbf{w}_e^a, \mathbf{w}_e^b, \mathbf{w}_{e+1}^a, \mathbf{w}_{e+1}^b) = \mathbf{w}_e^a \mathbf{w}_e^b \oplus \mathbf{w}_{e+1}^a \mathbf{w}_e^b \oplus \mathbf{w}_e^a \mathbf{w}_{e+1}^b \oplus \mathbf{r}_j$

$e = 1, 2, 3$

Experiments for ZKBoo

	SHA-1		SHA-256	
	Serial	Paral.	Serial	Paral.
Prover (ms)	31.73	12.73	54.63	15.95
Verifier (ms)	22.85	4.39	67.74	13.20
Proof size (KB)	444.18		835.91	

Soundness error: 2^{-80}
(137 repetitions of ZKBoo with soundness 2/3)

SHA-1 $\rightarrow$ 11680 AND gates
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- ... has a really cute name!!! :)

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Thanks for the attention! Questions?